Gukov

Knot ~~homologies~~ polynomials

$\swarrow$ $\searrow$

topological string theory — gauge theory

Oogari-Vafa — 3D Chern-Simons-Witten

Knot homologies

$\swarrow$ $\searrow$

refined/equivariant curve counting — 4D gauge theory

(comparison) — (skein rel.)

Gromov-Witten invariants

$\phi : \Sigma_g \longrightarrow X$: Calabi-Yau 3-fold

$GW_{g,\beta}$: defined through intersection theory on

$\mathcal{M}_g(X,\beta)$ = moduli space of stable maps

s.t. $\beta = \phi_*[\Sigma] \in H_2(X,\mathbb{Z})$

Def. $GW_{g,\beta}(X) = \int_{[\overline{\mathcal{M}}_g(X,\beta)]^{vir}} 1$

properties :
- $GW_{g,\beta} \in \mathbb{Q}$
- $GW_{g,\beta}$ can be computed by localization X^{G} Group
  mirror symmetry, ...

* Gromov-Witten invariants via gauge theory

X : symplectic 4-mfd

$Gr(X) = SW(X)$ (Taubes)

embedded surfaces — both integers

$Gr(X) \longleftrightarrow GW(X)$

relation

[MNOP]

X : Calabi-Yau 3 fold $GW(X) \longleftrightarrow DT(X)$

Donaldson-Thomas $\in \mathbb{Z}$

(moduli space of ideal sheaves

- open GW invariants

X : Calabi-Yau 3-fold

$\mathcal{L} \subset X$ lagrangian (special)

$$\phi : (\Sigma_g, \partial) \longrightarrow (X, \mathcal{L})$$

table

	Rational (maps)	Integers (gauge theory)	Refinements
closed 	GW stable maps	DT inv. ideal sheaves	Nekrasov (equivariant)
open 	open GW	Ooguri-Vafa	triply-graded invariants

X : Calabi-Yau total space of $\mathcal{O}_{\mathbb{P}^1}(-1) \oplus \mathcal{O}_{\mathbb{P}^1}(-1)$

(generic $\mathbb{P}^1 \subset$ Calabi-Yau, a n.b.d

$\mathcal{L}_K$: lagrangian submfd (noncpt) (conormal in T^*S^3 & deform

knot $K \rightsquigarrow \mathcal{L}_K$ (Taubes)

$$\underbrace{H^i(\mathcal{M}_g(X, \mathcal{L}_K))}_{\dim} = D_{Q,g,i} \in \mathbb{Z}$$

NB. This is, in fact, defined via BPS states

Q --- analog of β

$$\in H_2(X, \mathcal{L}_K) = \mathbb{Z}$$

generating function : $\overline{\mathcal{P}}(a, q, t) = \sum_{Q,g,i} D_{Q,g,i}\, a^Q q^g t^r$

a finite polynomial $\mathcal{P} \in \mathbb{Z}[a^{\pm}, q^{\pm}, t^{\pm}]$

Conj. For suffiently large N

(unreduced version) $\overline{KHR_N}(q, t) = \frac{1}{q - q^{-1}} \overline{\mathcal{P}}(a = q^N, q, t)$

$$\left(\overline{P_N}(q, t) \text{ of unknot } = \frac{q^N - q^{-N}}{q - q^{-1}} \right)$$

(reduced version) $P_N(\text{unknot}) = 1$

$KHR_N(q, t) = \mathcal{P}(a = q^N, q, t)$ for sufficiently large N

$\mathcal{P}(a,q,t) =$ Poincaré polynomial of a triply-graded homology theory

$$\mathcal{H}^{\mathrm{HOMFLY}}_{i,j,k}$$

$$\begin{tikzcd}
\mathrm{HOMFLY} \arrow[d, "a=q^N"'] & & \mathcal{H}^{\mathrm{HOMFLY}}_{i,j,k} \arrow[ll, "\text{Euler char.}"', "\text{i.e. } t=-1"] \arrow[d] & \cdots & d_N \ (N \in \mathbb{Z}) \\
P_N(q) & & sl(N) \text{ knot homology} \arrow[ll, "\text{Euler char.}"', "t=-1"] & &
\end{tikzcd}$$

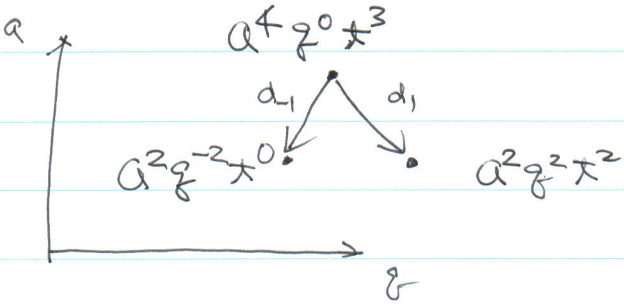
large N duality から来る.

<u>Conj</u>: $\exists$ triply-graded theory $\mathcal{H}^{\mathrm{HOMFLY}}$ comes with d_N s.t.

- $\chi(\mathcal{H}^{\mathrm{HOMFLY}}) = \mathrm{HOMFLY}$
- $d_N d_M = - d_M d_N$
- finite support $\quad \dim \mathcal{H}^{\mathrm{HOMFLY}} < \infty$
- specialization $\quad (\mathcal{H}^{\mathrm{HOMFLY}}, d_N) = \begin{cases} \text{KHR} & N \geq 2 \\ \text{Lee's theory} & N = 1 \\ \text{HFK} & N = 0 \end{cases}$
- canceling differential $\quad d_1, \ d_{-1}$

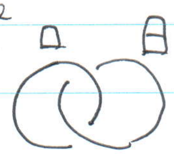
$$\dim(\mathcal{H}^{\mathrm{HOMFLY}}, d_1) = \dim(\mathcal{H}^{\mathrm{HOMFLY}}, d_{-1}) = 1$$

Ex, trefoil



N.B. Any represen

Ex, unreduced $\mathfrak{sl}(3)$ homology of the Hopf link ($\square$, $\boxminus$)



$$HSL^{N=3}_{\square,\boxminus}(q,t) = 1+2q^2+2q^4+q^6+t^2q^6+t^2q^8+t^2q^{10}$$

bdry cond.
をかえる.

ABCD of Matrix Factorization

$W_{\mathfrak{sl}(N),\square} = x^{N+1}$ $\qquad \delta W = x^{M+1} \rightsquigarrow d_M$

$W_{SO(N),S} = x^{N-1} + xy^2$ (spinor) $\qquad$ ($\delta W = y^2$ universal w.r.t. N

$W_{E_6,27} = z_1^{13} - \frac{25}{167} z_1 z_4^3 + z_4 z_1^9$

Symmetric of $\mathfrak{sl}(N)$ $\square\square$ $\qquad \sum (-1)^N t^{N+2} W_{\mathfrak{sl}(N),\square\square}(z,w)$

$$= (1+\cancel{t}z+tw^2)\log(1+tz+t^2w)$$